DOCUMENT RESUME

ED 242 531 SE 044 287

AUTHOR Stanic, George M. A.
TITLE The Growing Crisis in Mathematics Education in the Years Leading up to World War II.
PUB DATE Apr 84
NOTE 60p.; Paper presented at the Annual Meeting of the American Educational Research Association (68th, New Orleans, LA, April 23-27, 1984).
PUB TYPE Viewpoints (120)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS *Educational Change; *Educational History; *Educational Philosophy; Humanism; Learning Theories; *Mathematics Curriculum; *Mathematics Education; *Mathematics Instruction
IDENTIFIERS Mathematics History

ABSTRACT

The ways in which the "internal" reform ideas of mathematics educators in the period 1890-1940 interacted with the "external" reform ideas present in the general curriculum field are discussed. Using the interest groups identified by Kliebard (i.e., humanists, developmentalists, social efficiency educators, and social meliorists), the growing crisis in mathematics education in the years leading up to World War II is described, with extensive quotations from leaders of the day. It is suggested that the response of mathematics educators to the interaction of external and internal reform ideas provides valuable perspective for present-day mathematics educators. Based on this perspective, responses of mathematics educators to the present "crisis" in education are evaluated. (Author/MNS)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED242531

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

✓ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

The Growing Crisis in Mathematics Education in the Years Leading up to World War II

George M. A. Stanic
University of Georgia

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

George M. A. Stanic

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE044287

Abstract

The ways in which the "internal" reform ideas of mathematics educators in the period 1890-1940 interacted with the "external" reform ideas present in the general curriculum field are discussed. Using the interest groups identified by Kliebard (i.e., humanists, developmentalists, social efficiency educators, and social meliorists), the growing crisis in mathematics education in the years leading up to World War II is described. It is suggested that the response of mathematics educators to the interaction of external and internal reform ideas provides valuable perspective for present-day mathematics educators. Based on this perspective, responses of mathematics educators to the present "crisis" in education are evaluated.

The Growing Crisis in Mathematics Education in the Years Leading up to World War II

A crisis in mathematics education with its roots in the 1890s and its dénouement in the 1950s became widely recognized by mathematics educators in the years leading up to World War II. In 1930, William Betz suggested that "there probably has never been a time of greater doubt as to . . . [the] educational significance [of secondary mathematics]" (1930b, p. 104). In 1932, as president of the National Council of Teachers of Mathematics, Betz warned that "A passive attitude will only aid the negative propaganda emanating from the ranks of the educational iconoclasts and self-styled apostles of the 'new education'" (p. 301). As the decade of the thirties progressed, others spoke of mathematics being a "forgotten subject" (Glover, 1934), the "desperate situation" resulting from the public's acceptance of "the pronouncements of mathematical ignoramuses" (Bell, 1935, pp. 559-560), "the offensive action against mathematics in the secondary schools" (Hart, 1935, p. 72), the need for "some evangelism" (Hartung, 1935, p. 82), "the crisis that threatens . . . [the] extinction [of mathematics] as a required subject in major curricula of secondary schools" (Hedrick, 1936, p. 112), the need for mathematics educators to "face the facts and make appropriate adjustments" (Douglass, 1937, p. 62), the need for more "cooperation between high school and college teachers of mathematics" in order to fight the critics (Kempner, 1938), and "the expediency of compromise" (Maddox, 1938). Furthermore, in addition to all of the individual claims about and responses to

the crisis, the National Council of Teachers of Mathematics was sufficiently concerned about the situation to devote three entire yearbooks--Mathematics in Modern Life (NCTM, 1931), Mathematics in Modern Education (NCTM, 1936), and The Place of Mathematics in Secondary Education (NCTM, 1940)--to the defense of mathematics, generally, and its place in the school curriculum, specifically. The responses of the contributors to these yearbooks were not all the same, but the fact of their cooperative and organized response is further evidence of a general feeling of concern in the mathematics education community.

I will argue in this paper that this crisis can be described using the interest-group framework suggested by Herbert Kliebard (1981) and that it can be at least partially explained as the result of a complex interaction of "external" and "internal" reform ideas. Finally, based on the perspective gained from an (admittedly incomplete) understanding of this crisis, I will suggest some tentative ideas about the present (admittedly different) crisis in mathematics education.

II

In the concluding chapter of the 1970 NCTM yearbook on the history of mathematics education, Phillip Jones claims that "the real issues in mathematics education are continuing over time, changing slowly if it all":

> One of the lessons of history is that much about human beings changes only very slowly. Consequently activities, such as education, that are closely involved with the minds, drives, and emotions of human beings, change very slowly in their basic issues. In fact, the perspective of history often

shows that what were thought to be significant changes in issues were more nearly mere variations in the relative emphasis or prominence accorded them. (Jones, 1970, p. 455)

This commentary on the history of mathematics education is very much related to Herbert Kliebard's ideas about the development of the elementary and secondary school curriculum during the twentieth century, and these ideas provide the basic framework for this discussion. Kliebard (1981) characterizes mental discipline as representing "curriculum orthodoxy in the nineteenth century" (p. 2). With the decline in the influence of mental discipline, the field of education moved into the twentieth century with four interest groups emerging, each of which hoped to determine curriculum policy in the years ahead. Kliebard labels these four groups as the humanists (still, in some ways, linked to the mental disciplinarians), the developmentalists, the social meliorists, and the social efficiency educators. Led by Charles William Eliot and William Torrey Harris, the humanists emphasized the importance of the traditional disciplines of knowledge as embodiments of the Western cultural heritage. They were pitted against three reform groups, one of which, the developmentalists, was led by Granville Stanley Hall, who argued that the curriculum should be based on the natural order of development in the child. The social meliorists, with roots in the writings of sociologist Lester Frank Ward, saw the school as an important instrument for improving society, in general, and the plight of suffering individuals, in particular. Finally, impressed with the studies conducted by Joseph Mayer Rice, the social efficiency educators advocated a "scientific" approach to developing a curriculum which

would contribute to a smoothly functioning, efficient society, a curriculum through which each person would be prepared for his or her predetermined place in that society.

Jones's claim that the real issues in mathematics education change slowly, if at all, over time fits well with Kliebard's notion of the ongoing struggle between the four interest groups:

> The twentieth century was to become the arena where these four versions of what knowledge is of most worth and of the principal functions of schooling were presented and argued. No single interest group ever gained absolute supremacy, although general social and economic trends, the national mood, and local conditions and personalities affected the strength of each of these influences in different periods as the twentieth century progressed. In the end, what became the American curriculum was not the result of any decisive victory by any of the contending groups, but a loose, largely unarticulated, and not very tidy compromise. (Kliebard, 1981, p. 23)

Kliebard describes the 1890s as crucial in the development of these interest groups. In fact, the 1890s can be seen as a "crucible for curriculum change":

> By the 1890's, nineteenth century society with its reliance on the face-to-face community was clearly in decline, and with the recognition of social change came a radically altered version of the role of schooling. No longer was the school the direct instrument of a visible and unified community. Rather, it became a mediating institution between the family and an increasingly distant and impersonal social

order, an institution through which the norms and ways of surviving in the new industrial society would be conveyed. Traditional family life was not only in decline; even when it remained stable, it was no longer sufficient to initiate the young into a complex and technological world.

With the change in the social role of the school came a change in the educational center of gravity; it shifted from the tangible presence of the teacher to the remote knowledge and values incarnate in the curriculum. By the 1890's, the forces that were to struggle for control of the American curriculum were in place, and the early part of the twentieth century became the battleground for that struggle. (Kliebard, 1981, p. 1)

In this "crucible for curriculum change" can be found the roots of the crisis in mathematics education that came to be widely recognized in the 1930s. The humanists were the defenders of tradition, and their hold on the curriculum was coming under attack. But they remained the most highly developed interest group at the turn of the century. According to the humanists, mathematics should be taught because it is an important part of our Western cultural heritage and (at least for some of the humanists) because of its unique contribution to the development of an individual's reasoning ability. The developmentalists, the social efficiency educators, and the social meliorists did not reject the teaching of mathematics; but, when compared to the humanist justification, the justifications for teaching mathematics suggested by these three reform interest groups

threatened somewhat the traditional place of mathematics in the school curriculum.

For the developmentalists, all subject matter became of secondary importance as the focus moved from bodies of knowledge to the individual student. According to the developmentalist justification, mathematics should be taught only insofar as developmental experience shows it to be necessary; and the extent of an individual's interest and native endowment are the fundamental criteria in determining the extent to which a particular individual should study mathematics.

The social efficiency educators clearly recognized the importance of mathematics to our society. Indeed, even David Snedden, the most outspoken opponent of the humanists in the years leading up to World War II, recognized this and saw the need for some rigorous courses in mathematics to be offered in high school. Unlike the humanists, however, Snedden believed that only those students who would obviously need mathematics during their future vocations should take such courses; in fact, Snedden saw no need for required mathematics beyond sixth grade arithmetic.

The social meliorists, the least developed interest group at the turn of the century, did not pose the same threat to the humanists as did the developmentalists and the social efficiency educators. The social meliorist goals of equal opportunity and the fair distribution of extant knowledge are much more clear than the content of the school curriculum which would help achieve such goals. Given that Lester Frank Ward, the archetypical social meliorist, did not seem to think too highly of mathematics or

mathematicians, one cannot be too certain of exactly where mathematics would fit in the social meliorist program.

Recognizing the difficulty of labelling certain individuals as curriculum generalists--some of whom have interests in mathematics--and others as predominantly mathematics educators, there was at the turn of the century--however problematic the fine distinctions may be--a growing body of specialists in mathematics education who had particular views as to why mathematics should be taught. And, although there was certainly a diversity of viewpoint present among these mathematics educators, there are certain similarities or basic tendencies that characterized them. In short, we now want to ask: With which interest groups did mathematics educators who were establishing mathematics education as a distinct professional field of study in colleges and universities tend to affiliate?

One would expect that the basic tendency of these early mathematics educators would be to affiliate themselves with the defenders of the traditional curriculum--i.e., the humanists. As exemplified in the work of Jacob William Albert Young and David Eugene Smith, this is generally the case. Both Young and Smith saw mathematics as a vital part of our cultural heritage and as a vehicle for the development of reasoning power.

The problem with a scenario that characterizes mathematics educators as defending tradition and, therefore, affiliating with the humanist interest group is that it leaves the impression that both humanists, generally, and mathematics educators, particularly, represent stagnant and unchanging ideas. This is clearly not the case. Just as this era was filled with innovative

ideas of humanists such as Charles W. Eliot, ideas which were a fundamental part of the change taking place in the American curriculum, the 1890s and the turn of the century represent a significant era in mathematics education in terms of the issues being raised and the ideas being discussed.

Being in their general viewpoint very much in line with the central themes of Eliot and the Committee of Ten, the reports of the conference on mathematics of the Committee of Ten--published with the Committee of Ten report (NEA, 1893)--and of the committee appointed by the Chicago Section of the American Mathematical Society--published with the report of the Committee on College-Entrance Requirements (NEA, 1899)--were not uncontroversial. (See, for example, on the report of the conference on mathematics of the Committee of Ten, Greenwood, 1894 and Seerly, 1894.) Both reports embodied criticism of what they saw as traditional mathematics teaching, and both suggested changes. As an example of what was being discussed, consider the conclusions of the committee appointed by the Chicago Section of the American Mathematical Society, a committee chaired by J. W. A. Young:

1. To the close of the secondary-school course the required work in mathematics should be the same for all pupils.

2. The formal instruction in arithmetic as such should terminate with the close of the seventh grade.

3. Concrete geometry should be a part of the work in arithmetic and drawing in the first six grades.

4. One-half of the time allotted to mathematics in the

seventh grade should be given to the beginning of demonstrative geometry.

5. In the eighth grade the time allotted to mathematics should be divided equally between demonstrative geometry and the beginning of algebra.

6. In the secondary school, work in mathematics should be required of all pupils thruout each of the four years of the course.

7. Wherever, from local conditions, it is necessary to defer the beginning of geometry and algebra to the secondary school, here, likewise, geometry should be begun before algebra.

8. When once begun, the subjects of geometry and algebra should be developed simultaneously, in so far, at least, that both geometry and algebra should be studied in each of the four years of the secondary-school course.

9. The unity of the work in mathematics is emphasized, and the correlation and interapplication of its different parts recommended.

10. The instruction should have as its chief aim the cultivation of independent and correct thinking on the part of the pupil.

11. The importance of thoro preparation for teachers, both in mathematical attainments and in the art of teaching, is emphasized. (NEA, 1899, p. 148)

Now, these ideas are neither original nor revolutionary; but they cannot be fairly considered conservative given the context of the time in which they were written. The controversy embedded in

certain of the recommendations has persisted throughout the twentieth century.

Furthermore, with the work of, among others, Felix Klein in Germany, John Perry in England, and Eliakim Hastings Moore in the United States, the emerging field of mathematics education was presented with an agenda of significant issues to consider. During the latter part of the nineteenth century, Klein emphasized in his writings the relationship between the various areas within mathematics, a relationship embodied in the function concept. Klein also emphasized teaching mathematics by moving from the concrete to the abstract and by using applications of mathematics, though he warned that

> It is possible that through the mere mass of interesting applications, the real logical training may be crippled, and under no circumstances may this happen, for then the real marrow of the whole is lost. Hence: We desire emphatically, an enlivening of instruction in mathematics by means of its applications, but we desire also that the pendulum which in earlier decades perhaps swung too far in the abstract direction, should not now swing to the other extreme, but we wish to remain in the just mean. (quoted in Young, 1903, p. 54)

In England, Perry (1902) stressed the usefulness of mathematics, though it must be understood that Perry's conceptions of the usefulness of mathematics ranged from its role in "producing the higher emotions and giving mental pleasure" through "the aid given by mathematical weapons in the study of physical science" to "giving to acute philosophical minds a logical counsel

of perfection altogether charming and satisfying," with a variety of other examples of usefulness in between (see Perry, 1902, pp. 4-5); in short, it was quite a broad conception of usefulness. He criticized "advocates of orthodox methods" who seemed "willing to sacrifice every form of usefulness of mathematics to one for . . . the emotional or soul-preserving mind-training inherent in a perfect logical system; a huge complex deduced logically from simple fundamental truths" (p. 12) and advocated "assuming more complex things to be true" so that students "may know of all the modern discoveries" and use them sooner than they otherwise would (p.13). "Why not," asked Perry, "put aside ever so much more, as to let a young boy get quickly to the solution of partial differential equations and other useful parts of mathematics that only a few men now ever reach?" (p. 13). Interestingly, he also stressed that "the proper method of teaching any subject is through some kind of experimental work" (p. 19) and that teachers should "refrain from prosing and the setting of tasks" and instead "merely make timely suggestions and answer . . . questions," leaving the student to "find out things for himself" (pp. 23-24). In addition to his stress on students "making discoveries" (p. 8), Perry emphasized the importance of applications of mathematics and the relationship between mathematics and natural science.

E. H. Moore did a great deal to advance in the United States ideas like those of Klein and Perry. In his presidential address delivered before the American Mathematical Society on December 29, 1902, Moore (1902/1926) characterized the "fundamental problem" as "the unification of pure and applied mathematics" (p. 45). He asked: "Would it not be possible for the children in the grades

to be trained in power of observation and experiment and reflection and deduction so that always their mathematics should be directly connected with matters of thoroughly concrete character?" (p. 45). "Would it not be possible to organize the algebra, geometry and physics of the secondary school into a thoroughly coherent four years' course?" (p. 47). And Moore believed that Perry was "quite right in insisting that it is scientifically legitimate in the pedagogy of elementary mathematics to take a large body of basal principles instead of a small body and to build the edifice upon the larger body for the earlier years, reserving for later years the philosophic criticism of the basis itself and the reduction of the basal system" (pp. 47-48). Based on these ideas, Moore advocated a "laboratory system of instruction in mathematics and physics, a principal purpose being so far as possible to develop on the part of every student the true spirit of research, and an appreciation, practical as well as theoretic, of the fundamental methods of science" (p. 49).

Given that the movement toward "unification" of the areas within mathematics and of mathematics with the sciences found its greatest strength in and around the University of Chicago, it is not surprising that Young would be more sympathetic than Smith to the ideas of people like Klein, Perry, and Moore (see, for instance, Young, 1903; 1906, pp. 87-121), although Young (1906) did warn that "It would be doubtful gain, even if it were possible, to abolish the distinction between algebra and geometry, mathematics and physics, when breaking down the wall of separation between them which has often been regarded as an insuperable

barrier" (p. 102). It appears that Smith was a bit more reluctant to accept some of the ideas expressed by Klein, Perry, and Moore, though he did not completely reject their ideas. In his response following Perry's address, Smith commented that "There is a tone running through the address that grates on one who loves mathematics for its own sake; but tones are intangible, and it is difficult to tell where this one has its origin. It is a tone that seems to deride the love of mathematics per se. . . . Certain it is that the speaker told a great deal of truth, and if he went to an extreme, it was because in fighting conservatism one must sometimes be ultra-radical" (in Perry, 1902, p. 91).

Smith responded a bit more boldly to some of the issues in an article written in 1905 entitled, "Movements in Mathematical Teaching." Referring to Moore's 1902 address, Smith claimed that "It was a fact apparent to all who heard it that the address was not favorably received by many of those present" (p. 135). After describing the response of educators in "the East" to Moore's ideas, Smith came to the following conclusion:

> In conclusion, the feeling in the East, as I interpret it after much study, is one of earnest desire to make the class in mathematics a workshop, but in the high school it must be a workshop in pure mathematics. In arithmetic, where the field of real application is very large, there is a manifest desire to banish those problems that pretend to relate to business and science, but do not do so, to substitute genuine ones in their stead, but to hold to such an amount of abstract work as to bring that mechanical efficiency that is necessary to fairly rapid and accurate computation. In

> algebra and geometry, where the field of real applications is very small, the chief effort will be to turn the class into a laboratory in pure mathematics, recognizing that the applied problem shall be used as one of several means for arousing interest, but that this is a mere incident. (pp. 138-139)

Seemingly describing himself as much as his area of the country, Smith claimed that "while the East thinks slowly it thinks judicially, while it works conservatively it works thoroughly, while it is not given to rash experiment it is not blind to results, and while it is influenced by traditions it is ready to change when it sees other plans proved to be better then [sic] its own" (p. 139).

As was true in the case of the ideas presented by the mathematics conference of the Committee of Ten and the Committee appointed by the Chicago Section of the American Mathematical Society, the ideas of Klein, Perry, and Moore were not new; but the state of the American curriculum was such that ideas like those described above became legitimate and significant foci of discussion for people concerned about mathematics education. So, even though it seems reasonable to characterize mathematics educators like J. W. A. Young and David Eugene Smith as being affiliated with the humanist interest group in terms of how the place of mathematics in the school curriculum was justified, it would not be reasonable to characterize the views of mathematics educators as stagnant and unchanging, just as it would be unreasonable to characterize the views of humanists, generally, in this way. Furthermore, both Young and Smith were affected by the

other interest groups, particularly the developmentalists; and this impact is reflected in their writings. Interestingly, Smith even wrote an article entitled, "Mathematics in the Elementary School" in 1903 with Frank M. McMurry, a colleague of Smith at Teachers College and one of the early members of the Herbartian movement (a movement which, in some sense, merged with or, perhaps more accurately, developed into the developmentalists). In the article, Smith and McMurry stressed that "the child's interest in the quantitative side of life should be the highest immediate aim of the teacher of mathematics in the grades. . . . [A]nd the nature of the child, together with the needs of society, should constitute the main standard in selecting subject-matter. This signifies that these two ideas should be given a fuller control in mathematics than has thus far been allowed" (p. 94). However, in a later piece which was described in its preface as necessary because the 1903 article with McMurry was out of print, Smith (1909b), writing alone this time, cautioned his readers about the "tendency throughout the country to make arithmetic, as other subjects, more interesting to children":

> Our lesson, therefore, is that we should do all in our power to make arithmetic interesting and even attractive to the children, but that we must not hope to attain this result by offering a sickly substitute for the vigorous subject that has come down to us. Unfortunately, we have not been free from this fault of making our arithmetic, and particularly our primary arithmetic, anaemic. (p. 39)

The 1909 article indicates that the impact of the developmentalists on Smith was not as great as a reading of only

the article written with McMurry might lead one to believe. In looking back at any person's work, it is difficult to characterize accurately his or her ideas. We often expect a great deal, perhaps too much, from people in terms of absolute consistency across the years; and it is extremely difficult to decide when a particular piece is inconsistent or, instead, an unusual, but reasonable, extension of a person's ideas. In this particular case, Smith was kind enough to remind us of his basic humanist orientation in a follow-up to an article that might have been difficult to interpret standing on its own.

For mathematics educators, then, defending the place of mathematics in the school curriculum and affiliating for the most part, at least in this era, with the humanists is not to be considered synonymous with unchanging ideas about the curriculum . The crucial point is that even when change is suggested by those defending a particular tradition, it is change bounded by certain "givens"; and in the case of mathematics educators, the basic given is that mathematics should be an important part of the school curriculum.

III

All the interest groups recognized the importance of mathematics to our society, and all agreed that everyone needed to study arithmetic. However, there was little agreement beyond this; different justifications for teaching mathematics have different implications for the issues of who should be required or encouraged to study how much mathematics and what the form and content of the curriculum should look like. With such issues to motivate debate, the 50-year period from 1890 to 1940 was

characterized by a complex reform milieu in which "external" (in the general curriculum field) and "internal" (in mathematics education) reform movements interacted to create a variety of ideas within and among the interest groups. The groups overlapped in some of their ideas; and coalitions formed, the most significant coalition being that between the developmentalists and the social efficiency educators. Individuals used the same language to talk about different things, different language to talk about the same things; and just as each interest group evolved and changed, so did each individual who made up those groups.

Throughout all this, an overriding concern of the various historical actors seems to have been what to do about the clear changes in the quantity and perceived changes in the quality of the school population. This presented perhaps the greatest challenge to the humanist interest group, the defenders of the traditional school curriculum, a group whose ideas had their roots in a fundamentally optimistic faith in human intelligence. Recognizing that individuals are different from each other in intellectual capability, humanists nonetheless argued that most individuals can handle the challenge of a rigorous education. A form of individualization was also part of the roots of the humanist interest group; but, as espoused by Charles W. Eliot, it was individualization based primarily upon individual interest and choice, individualization which, in fact, further exemplifies Eliot's faith that human beings can and should be challenged and trusted to made decisions for themselves. Not all humanists expressed the same faith that Eliot did, and not all agreed with

his elective system. But most humanists at least believed that what is good for some is good for all when it comes to making decisions about what should be taught.

Developments during the 50-year period clearly challenged that humanist faith. The educational world of the humanists was changing most drastically, and they responded to the challenge and the change in a variety of ways, ways that sometimes led particular individuals away from the basic humanist viewpoint. Some attempted to document further the vital role of mathematics in our society and warned that those who foolishly neglected to study it would have a hard time making it up when they found it necessary for a particular career. Others went so far as to suggest that our national security required that more people study more rigorous mathematics. And still others attempted to use the language of the external critics to advocate reforms with their own history inside mathematics education.

The situation during this 50-year period was not simply one of mathematics educators being "attacked" by a group of educators not closely associated with mathematics education. For example, Guy Wilson, who wrote for the social-efficiency-oriented Committee on Economy of Time, continued to focus his research and writing on mathematics education; any attempt to label him as a member of a group of general educators not associated with mathematics education is problematic. Furthermore, even apart from people like Wilson who expressed a growing interest in mathematics education, much of the push for reform came from people who were clearly part of the mathematics education community. For instance, writing in the Mathematics Teacher in 1909, N. J. Lennes

(who considered himself a mathematician and who had a strong interest in mathematics education) combined the humanist faith in the value of the study of mathematics with the developmentalist concern for the child in an interesting article on the teaching of algebra. After discussing principles and characteristics of "modern pedagogy," including the importance of "the study of the child itself,--its needs, interests and capacities" (p. 95), Lennes tried to explain the reason for "discontent" about the teaching of secondary-school mathematics:

> It is difficult to assign reasons why the teaching of secondary mathematics has adjusted itself more slowly to the requirements of modern pedagogy than that of any other great subject, but there can be little doubt that this exceptionally tardy conformity of practice to our best principles is the reason for the prevailing discontent. (p. 98)

His "fundamental proposition" was that "in the beginning we will as far as possible study only such algebraic processes as we find are needed in solving concrete problems. "That," said Lennes, "has been the universal rule in arithmetic and we now propose to continue it into the algebra" (p. 101). However, he continued, "while the beginning of the study of algebra should emphasize its dynamic rather than its logical aspect, it is by no means intended that it should end there. When the child has become familiar with the main processes of algebra through their dynamic aspect," the processes "then become legitimate objects of study for their own sake" (pp. 103-104). Lennes concluded his discussion with the

following comments on bridging the gap between the child and the mature person:

> Recognizing fully the great gap between the child and the mature person, it is our purpose to bridge this gap so that the transit may be normal and healthy. Beginning with the dynamic elements of algebra, which is everywhere the child's element, we learn to use its processes in doing things that the child sees some sense in doing. As the processes become familiar they themselves become objects of interest and are then studied. The great desideratum is that a natural interest is maintained all along. In this way the pupil is stimulated to greater and more pleasurable efforts. His real acquisition is much greater though he does not know so many special cases the first year. His whole intellectual life is healthier in that he is keeping in constant touch with his own common sense. (p. 104)

Again, both because of the difficulty in labelling certain reformers as being outside of mathematics education and because of examples of reform ideas on the part of people who were clearly part of the mathematics education community, the situation in the years leading up to America's involvement in World War II was not simply one of mathematics educators being "attacked" by "outsiders." However, and even more important, many mathematics educators viewed their efforts at "internal" reform (i.e., reform within mathematics education) as distinct from "external" reform (i.e., reform within the general curriculum field). In the words of William Betz (1908),

> That one has a right to speak of a reform movement in the teaching of mathematics, must be evident to any but a very indifferent observer. The agitation in favor of better teaching is not confined to our own country, and its effects are felt in colleges, universities, secondary and primary schools, with almost equal force. It is unnecessary to recite . . . all the details of its origin and progress. . . . It is sufficient to say that since Prof. E. H. Moore's memorable address in December, 1902, "On the Foundations of Mathematics," an ever increasing number of teachers have become actively interested in the pedagogy of mathematics. (p. 635)

Those mathematics educators who saw their reform efforts as distinct from reform within the general curriculum field were not blind to the changes going on around them; they were recognizing and stressing a legitimate tradition of reform in mathematics education. In effect, mathematics educators both helped create and were forced to respond to a complex interaction of reform ideas.

Some, like Betz, faced the situation with a reasonably open mind. While stressing the importance of the internal reform, the beginning of which he attributed to E. H. Moore's address, Betz attempted to place the reform movement in a wider context in his discussion of "transforming influences." He listed five such influences, including "modern industrialism" creating "a great outcry for more practical school work," a "high school population more diversified than ever before," "certain revolutions in school curricula and methods of instruction," the "new education," and

"research in the domain of pure mathematics" (pp. 626-627). His discussion of the "new education" is particularly interesting:

> Its fundamental precept that all work must be arranged psychologically and adapted strictly to the child's power of comprehension, is eminently sound. But it has also given us the enriched curriculum, and the doctrine of interest which replaces all objective standards by the subjective attitude of the child. Unquestionably this means at once a distinct advance and a very real source of danger. The complaint is not infrequent that in many cases the young are learning to depend too much upon the inspirational powers of the teacher, that all real difficulties are carefully avoided, and that the very aim of all true education, to develop a strong character and to create self-activity and initiative, is thereby defeated. (p. 627)

Betz also warned of "exaggerations" in the "new education" that "rob many otherwise excellent ideas of their legitimate influence and place upon them the stigma of the faddist" (p. 627).

Using ideas and terms similar to those of the most ardent developmentalists and social efficiency educators, Betz produced a different message, one based upon the assumption that "never has the unique importance of mathematics been more generally recognized, nor has there been more mutual sympathy and cooperation" (p. 633). For instance, he said about the geometry course that

> The one-sided theory of mental discipline in the elementary course must go. The over-insistence on discipline for its own sake has done incalculable harm to the interests of

mathematics. It is well that we are beginning to realize this. . . . Our civilization craves action, and a subject that loses contact with life is doomed. (p. 630)

He also asked: "Should we not eliminate all that is not absolutely essential?" (p. 631). Yet just as Charles W. Eliot, the humanist, had spoken of both "shortening" and "enriching" the grammar-school course, Betz sought to eliminate non-essentials so that "The time gained by such omissions could be devoted to more significant topics, and would enable us to introduce at least a few interesting and valuable applications" (p. 631). The crucial point is that Betz viewed suggested reforms with a basic faith in the value of teaching and learning mathematics; the same could not be said of some of the developmentalists and social efficiency educators whose terminology he sometimes used.

Although Betz continued to look at all suggested reforms with an open mind, it is clear that he gradually expressed more negative views about the external reform movements and more concern about the state of mathematics education. Like so many other educators, he continued to claim that "It is an open secret that our policy of extending every educational opportunity to the masses has brought about a further lowering of standards. This may be unavoidable, but it does not make the fact less serious" (Betz, 1913, p. 220). Although he felt that the "demand" for "efficiency" in the high school "should be welcomed enthusiastically by all teachers," he again warned of "loose thinking":

In many of the current efficiency arguments there is an element of charlatanism, of gross materialism, and of loose

thinking. Some efficiency experts would have us think that the work of a school can be gauged like the output of a factory. It should not be forgotten that the finest and most lasting results of good teaching cannot be tested by the methods of the counting-room or measured in dollars and cents, and in fractions of a second. (pp. 222-223)

Betz, in a sense, was attempting to distinguish between efficiency as the ability to use acquired knowledge and efficiency as economy of time and effort and the elimination of waste, supporting the former meaning and disputing the latter.

Unlike his comments in the 1908 article, Betz eventually even warned of the dangers of overreacting to mental discipline experiments. "Contrary to the usual impression conveyed by superficial alarmists," commented Betz in 1913, "it may be said confidently that 'we have neither proved nor disproved the formal discipline dogma'" (p. 224).

In his 1913 discussion of "the aims of mathematical teaching," Betz claimed (again using "efficiency" in the sense of the ability to use acquired knowledge) that the aims "center around the two ideas of insight and efficiency, which may also be described by the terms impression and expression, or by form and content. The one is subjective, the other objective." He then described the "five important aims of mathematical teaching," which he classified under symbolism, applications, the function concept, space intuition, and logic (p. 226). Betz concluded that

It is very doubtful whether any other high school subject can produce equally cogent and permanent reasons for being included in the curriculum. This becomes all the more

apparent if it is observed that each of these five aims is essential from either a purely ideal or a narrowly utilitarian standpoint. To borrow a happy phrase of Professor D. E. Smith, mathematics is to a remarkable degree "potentially practical." It may be said without hesitation that if the opponents of mathematics should ever succeed in abolishing it from the curriculum, public opinion would demand its restoration within a year. Mathematics meets a fundamental human need. Finally, let it be remembered that ours is an age of mental unrest and uncertainty. To acquaint the minds of the young at this critical period with reliable standards of intellectual certainty is a service no less real because it is often overlooked, a service that endows mathematics with the highest moral value. (p. 227)

Like many other mathematics educators, Betz continued to write with a spirit of compromise, saying that "The mathematical curriculum of the future must meet the wants of all types of students. For some of these types only actual experiment can determine the most desirable selection and sequence of topics" (Betz, 1913, pp. 229-230). But he also said that the "five aims cannot be fully realized unless every student in the general course is required to become familiar with the elements of algebra, of plane and solid geometry, and of plane trigonometry" (p. 230). In defending the latter claim, Betz, to a certain extent, turned the social efficiency educators' directly functional rationale back upon them:

If any one should consider this an unreasonable demand, let him make a brief study of the European requirements. If

> America wishes to compete with Europe along industrial, technical, or scientific lines, it cannot ignore the increased educational opportunities given to European secondary students. (p. 230)

Admitting that mathematics was important to our society as a whole, social efficiency educators nonetheless felt that this did not mean that all students had to study mathematics in depth. It was, they thought, essentially useless to the day-to-day activities of most people. Social efficiency required that only a few people study mathematics in depth. (One must remember here that even the definition of "in-depth study" would be different for a mathematician and a social efficiency educator.) However, in bluntly linking our future national security to the argument, Betz was suggesting that--even though it would be beneficial for each student as an individual--as many people as possible should take a reasonable amount of mathematics in the high school in order to make certain that each could play a role in the competition with other nations and that all potential mathematicians and scientists could be discovered. In one sense, it was an argument against the belief of certain social efficiency educators that future societal roles could be clearly predetermined. In another sense, it was an argument for a certain kind of "social efficiency."

While Betz was attempting to evaluate suggested reforms with an open mind, it appears as though some mathematics educators responded to the complex reform milieu by using the language of the developmentalists and social efficiency educators to advocate reforms which in fact had an early history in mathematics

education separate from these general movements; that is, they used external reform rhetoric to push for internal reforms that had a history of their own. The best example of this lies in the persistent attempts of certain mathematics educators to implement a "unified" mathematics curriculum in the schools. The extent to which this use of external reform rhetoric had problematic consequences for justifying the teaching of mathematics is an interesting part of the history of mathematics education.

Using adjectives like "correlated," "fused," "unified," and "parallel," certain mathematics educators, predominantly at or near the University of Chicago, advocated "breaking down the walls" between mathematics and other subject areas and between the various areas within mathematics itself. These educators were definitely not all arguing for the same thing; in fact, there were significant differences of opinion within this group (see Sigurdson, 1962). Articles by Breslich (1916, 1920, 1933, 1935); Breslich, Schorling, Wright, and Irwin (1916); Hawkes (1911); Myers (1907); Reeve (1915, 1920, 1922, 1924); Schorling (1926); and Young (1907, 1912) give some indication of the kinds of discussions taking place (though these by no means reflect all that was being said about "unified" mathematics). The crucial point for this discussion can be brought out be looking a bit more closely at what Reeve was saying.

In a 1916 article, Reeve spoke highly of H. C. Morrison's (1914) chapter in the Thirteenth Yearbook of the National Society for the Study of Education: Part I, a chapter which reflected a mixture of developmentalist and, particularly, social efficiency thought:

> No one who reads Mr. Morrison's article will feel that he is attempting to discredit the place of mathematics as a part of the child's educational equipment. On the other hand, his attitude is friendly and constructive--this, it seems, is one good reason why all mathematics teachers should read what he has to say. (p. 204)

Not all mathematics educators described Morrison's discussion in quite the same way as did Reeve; Minnick (1916), for instance, characterized Morrison's work as a "severe criticism against present practices in mathematics" (p. 81).

Reeve then mentioned criticisms of the teaching of mathematics made by David Snedden and Abraham Flexner. He did not accept all of Snedden's and Flexner's criticisms, but he did use their criticisms (and those of Morrison) to support his own views. In particular, he responded to Flexner as follows:

> It may not be a hard task for teachers of mathematics to convince Dr. Flexner that many of the faults of which he speaks are now apparent to the progressive teachers of mathematics and that in many cases the attempts to remedy existing conditions have been not only intelligent, but noteworthy and in some cases in line with nearly all the reform Dr. Flexner suggests. (p. 205)

Furthermore, Reeve said that he was in "perfect agreement" with "some of Dr. Flexner's statements." "It is to be hoped," said Reeve, "that mathematics teachers will not sit by and permit all of the criticisms in the future to be made by outsiders" (p. 206). In response to Flexner's belief that "the course in mathematics should include 'nothing for which an affirmative case can not now

be made out,'" Reeve replied: "This is exactly what teachers of mathematics who are really interested in the future are trying to do" (p. 207).

Reeve's answer to these critics, although he said it would not meet all of their "demands," was "some scheme of unifying high-school mathematics . . . keeping in mind all the while that increased efficiency in the technique of teaching must be looked after at all times" (p. 207). More specific examples of Reeve's attempt to use external reform rhetoric to further his own internal reform ideas can be seen in the description of his unified program:

> A decided advantage of such unification in the first year is that it gives the child a broader knowledge of the nature of the field of mathematics so that he is able to see better what lies in store for him if he goes on. And in the second place, <u>if he does not continue, he has what the writer thinks is a better asset from a social standpoint. This second item it seems is what most people are demanding of the subjects in the curriculum, viz., that they function socially</u>. (p. 209, emphasis added)
>
> In the third year the unification can be made as easily (so far as the subjects themselves are concerned) <u>and with as much added interest and value in economy of time and efficiency of effort</u>. (p. 211, emphasis added)

Reeve made a particularly interesting comment near the end of the article, a comment in which he more or less admitted that he was linking a reform movement with its own history in mathematics education with the reforms suggested by the "outsiders":

Finally let it be remembered that these suggestions are not considered as final, and that they are not given merely in the hope that something may be done to satisfy the cry for reform in the teaching of mathematics (which is in many respects legitimate) but to try to remedy some of the weaknesses of the system of which we were aware long ago, and also to increase the mathematical power of the pupils who go out from our classes so that their knowledge may function most efficiently. (pp. 211-212)

The reform milieu grew even more complex after the First World War. Some of the difficulty mathematics educators faced in responding to this milieu can be seen in the establishment of the National Council of Teachers of Mathematics in 1920. In the first issue of the Mathematics Teacher published by the National Council (it was previously the official journal of the Association of Teachers of Mathematics in the Middle States and Maryland), the following "reasons for the National Council" are provided:

First, it will at all times keep the values and interests of mathematics before the educational world. Instead of continual criticism at educational meetings, we intend to present constructive programs, by friends of mathematics. We prefer that curriculum studies and reforms and adjustments come from the teachers of mathematics rather than from the educational reformers.

Second, it will furnish a medium through which teachers in one part of the country may know what is going on in every other part of the country. Significant reports and studies and experiments will be given wide circulation

through the official journal. Otherwise they would be known only locally.

Third, the Council through its journal will furnish a medium of expression for all of the teachers of the country. Thus, a feeling of solidarity will be aroused. All teachers of mathematics will know that they are members of one family, working together under a common leadership.

Fourth, the Council will help the progressive teacher to be more progressive. It will also arouse the conservative teacher from his satisfaction and cause him to take a few steps ahead.

Fifth, the splendid work of the National Committee on Mathematical Requirements will be conserved and extended as time and new conditions may show the need.

Sixth, the Council should receive the support of every teacher of secondary mathematics because it intends to maintain a journal whose editors and writers are real class room teachers. In fact, the teachers themselves are the owners of the journal, if they will but join the Council. (Austin, 1921, p. 3)

The first and fourth reasons are particularly interesting and informative. The NCTM was, at one and the same time, leery of "educational reformers" and supportive of "progressive teachers." Yet, given the diversity of suggested reforms inside and outside of mathematics education, these reasons are not necessarily contradictory. It is especially interesting that Harold O. Rugg, who has been generally characterized as a "progressive," was the chairman of the committee that put together the first constitution

of the organization; and he also served as its first vice-president.

Fundamentally, the NCTM was formed in response to attacks on high-school mathematics. To quote again from the introductory article:

> Mathematics courses have been assailed on every hand. So-called educational reformers have tinkered with the courses, and they, not knowing the subject and its values, in many cases have thrown out mathematics altogether or made it entirely elective. . . . To help remedy the existing situation the National Council of Teachers of Mathematics was organized. (Austin, 1921, pp. 1-2)

It appears as though the NCTM defined "progressives" as supporters of mathematics and labelled their foes as "so-called educational reformers."

This acceptance of certain kinds of "progressives" (or, perhaps, this stipulation of a particular definition of "progressives") is not easy to interpret and is another example of what one might label the "language problem" in historical research. Quite simply, different people used the same terms in far different ways and with far different motives. For instance, just as Betz (1908, 1913) seemed to use the language of the external reformers in an attempt to address the reform milieu with an open mind, so did Eugene Randolf Smith (1927, 1929) of the Beaver Country Day School in Chestnut Hill, Massachusetts and George W. Myers (1918, 1921, 1922) of the University of Chicago do so. E. R. Smith merged a basic faith in the value of mathematics with a belief that teachers of mathematics had to respond in some

way to a changing society; he seemed to be particularly, but not exclusively, interested in developmentalist ideas. Though he was clearly one of the early advocates of "unified" mathematics, Myers' overwhelming concern was that mathematics educators respond in some way to the reform milieu. As he said in "Reaction vs. Radicalism in the Teaching of Mathematics" (1922):

> Let the radical who must be that or nothing be radical, for what we need is energy. The radical attitude is more helpful than that of the satisfied and complacent routinist. Let the reactionary who must be that or nothing be reactionary, for action is needed, and if he be a good sample, he is more likely to see the light of reform than is the complacent routinist. It takes energy to be either a reactionary or a radical, and energy will at least loosen things. Good counsel to the professional brethren would seem to be as follows: "Be an open-minded worker for reform if you can. If you cannot be this, be a radical. If you can be neither, be a reactionary." The one who cannot be any of these is in grave danger of gravitating into a complacent routinist, and thus become the least helpful of all. He many be inconspicuous and comfortable while his more courageous brethren are bearing the burden and the buffetings that his very type of conduct has brought on.
>
> May every one of us aspire to be a force in the reform now making, positive if we can be, negative if we must be, but still an active force. No real teacher dares look on the struggle with indifference. To do so means loss of professional standing. The stake in the game now going on

is nothing less than the alternative of a substantial reduction of the mathematical element in education to the vanishing point, or the marked improvement of quality of output of the element in the secondary schools. Can we not forego the privilege of bandying epithets for a season and roll up our sleeves for reorganization and reform? (pp. 145-146).

On the other hand (and although there is a fine line dividing open-minded response and the use of external reform rhetoric to advocate one's own internal reform ideas), just as Reeve had used the rhetoric of the external critics to advocate his own beliefs about unified mathematics in the years leading up to America's involvement in World War I, so did Reeve and other mathematics educators continue to do so in the years following World War I. Reeve, in particular, responded in different ways to different audiences. In a paper read at the meeting of the National Council of Teachers of Mathematics on March 1, 1920 (a paper which was published in the Mathematics Teacher in 1922), Reeve began as follows:

I shall not attempt, in this paper, to dis-credit our traditional methods of teaching algebra in the first year of the high school, followed by plane geometry in the second year, intermediate algebra in the third year, and so on. I say this in spite of the fact that much of our traditional practice and the accompanying results might justify one in so doing. In short, I am not interested in a destructive type of criticism of past methods with a view to setting up new bits of content (or at least

reorganized content) and technique of procedure. Certainly, I should not favor a method which would seem to be attempting to force any set program upon the teaching body. The best progress is not made in that way. With many teachers of mathematics, the traditional order of treatment, if not the traditional methods, will prevail. Moreover, this will be true even after much experience and available scientific data may make a trial of some form of reorganized content and methods seem wise and feasible. (Reeve, 1922, p. 381)

However, when addressing the Conference of Principals and Superintendents of the Minnesota Educational Association in April of 1920 (an address which was quite similar in many respects to that given at the NCTM meeting), Reeve began as follows:

For a long time mathematicians and others as well have occasionally denounced our methods of teaching mathematics and usually from their own point of view. Men like Sylvester De Morgan, Hamilton, Perry, Snedden, Morrison, and Flexner are examples of the kinds of critics we have had.

A careful study of the recent tirades against mathematics reveals criticisms really aimed at faulty organization and improper methods of teaching the subject matter, rather than a desire to discredit the subject itself. In short, people are saying that tradition alone is not a sufficient reason for giving the subject the prominent place it now holds in the curriculum and that even though the subject when properly taught may have values of extreme importance, we cannot continue to give so much time to its study unless we can show that it functions properly in the

student's life.

> There are a great many reasons why one who studies the present situation from an unbiased standpoint must have a great deal of sympathy with the last statement. There is not a person today who cannot pick out some phase of his mathematical training that has been of little real service compared with what might be expected from the time spent in acquiring that particular experience. There seems to be no need of discussing further the values to be obtained from the study of mathematics, for on most of these at least, people generally are agreed, but the greatest objection to be met is that we are not securing these values. (Reeve, 1920, p. 258)

In both cases, Reeve was advocating his program of general mathematics for the high school. In these two articles, it is clear that "general mathematics" was being used as an alternative label for many of the ideas inherent in the unified mathematics movement. In both addresses, he made the same basic proposal:

> What we propose to do, what we are already doing in the University High School at the University of Minnesota is to arrange all of these courses so that we shall not have courses in algebra, geometry, or trigonometry as such, but a definitely arranged and psychologically ordered course in mathematics. The plan which we are now formulating will give the student who desires to continue a four-year course in mathematics in the high school, which we hope will eventually be the equivalent of what is ordinarily done in the high school plus one or one and one-half years of college work.

> It is also hoped that the course will be more compact, will involve less waste, develop more power and produce even better results than we are getting from present methods. The content of this course will be algebra, plane and solid geometry, trigonometry, most of the analytic geometry, and some of the elements of the calculus. (Reeve, 1920, p. 262)
>
> My proposal is that wherever the conditions permit we arrange our mathematics so that we shall not have courses in algebra, in geometry, or in trigonometry as such, but a definitely arranged and psychologically ordered course in mathematics. The plan which we are now formulating will give the student who desires it a four year course in mathematics in the high school plus one or one and one half years of college work. It is also hoped that the course will be more compact, will involve less waste, develop more power, and produce even better results than we have been getting from traditional methods. The content of this course will be algebra, plane and solid geometry, trigonometry, most of the analytic geometry, and the fundamental elements of the calculus. (Reeve, 1922, p. 385)

It is also clear, however, that in the address before principals and superintendents, he was more willing to link his reform ideas with external rhetoric for reform. On the one hand, this does not seem unreasonable. Clearly, one must understand and adapt to one's audience. On the other hand, given Reeve's intent that his program "lead the child into much higher and more powerful mathematics without any ultimate loss" (1920, p. 260), it is important to consider whether Reeve's intent meshed well with

the external rhetoric he was willing to use. That is, one must consider the possible--but perhaps unintended--consequence that the rhetoric of the social efficiency educators, in particular, might interact with and change the substance of the suggested internal reform. And, although one would want to be careful not to make a direct causal claim, if one looks closely at the different and changing meanings given to "general mathematics," it at least indicates the possibility that the internal and external reforms interacted; the connotations that came to be associated with "general mathematics" differed from the rigorous-program connotation that Reeve advocated in his two addresses.

For instance, in the first yearbook of the National Council of Teachers of Mathematics, Raleigh Schorling (1926) said the following about general mathematics:

> General mathematics is an introductory, basic, exploratory course in which the simple and significant principles of arithmetic, algebra, geometry, intuitive geometry, statistics and numerical trigonometry are taught so as to emphasize their natural and numerous interrelations. There has been some confusion of terminology. Few people, for example, would venture to draw a distinction between general mathematics and correlated mathematics. However, there are at least two important differences. First, that general mathematics is not a desirable substitute for demonstrative geometry, and second, that the value of general mathematics depends upon what is correlated and how it is correlated, rather than upon the fact that something has somehow been correlated. It is argued, moreover, that the movement

> started by Professors Moore and Myers put far too much emphasis upon correlation for the sake of correlation. General mathematics has used the correlation idea,--the emphasis upon interrelations. But its chief objective has been wholly different from that of correlated mathematics. The aim of general mathematics is close to that expresssed in the document "Cardinal Principles of Secondary Education." The movement, started about ten years ago, represented an effort to get a course in the ninth year which would more nearly meet the needs of pupils, particularly those of low ability and poor background and those who would leave school early before graduating. There was no intention to set up three or four years of college preparatory courses. (pp. 100-101)

Associating general mathematics with the NEA-sponsored report, published in 1918, entitled, Cardinal Principles of Secondary Education does not seem to fit with Reeve's discussion of the rigorous general mathematics program in his two addresses. Although social-efficiency-standard-bearer David Snedden (1919a) felt that "failure to utilize what . . . [were] available as sociological guide posts . . . led the committee [that wrote the Cardinal Principles report] into unnecessary confusion" (p. 520), he did find reason to commend the report; and it is clear that the report reflected much of the social efficiency ideal that was coming to dominate thought in education.

Even Reeve (1924) came to advocate a form of general mathematics which might be described as almost a "remedial" course for those who could not handle rigorous mathematics:

If it turns out that there are some pupils who cannot learn mathematics, or who profit to a very small degree by taking it, then I should not require it of them. If I had my way, I should require mathematics only in the junior high school. The joy that would come to teachers and pupils alike by having in the tenth, eleventh, and twelfth year classes only the pupils who love mathematics so that they take it by choice would be a great blessing to all concerned. Moreover, I am confident that we mathematics teachers would still have classes to teach.

Naturally, the question of what to do with pupils who are excused from certain courses will arise. If they are to take mathematics at all, they should be given a course with which they can succeed. Such a course can be worked out and has been worked out in some schools by experiment. I believe that some modified form of general mathematics would meet the needs and interests of such pupils. (pp. 451-452)

It is not as though this represented some overnight change in Reeve. In his 1920 address before the principals and superintendents, for instance, he expressed a concern about students "forced to leave the high school at the end of one or two years" and urged mathematics educators to use their knowledge of "individual differences in ability":

The purpose of a general mathematics course in the high school is to furnish the basis for a modern scholarly course in elementary mathematics that will give such careful training in power and appreciation as well-informed citizens of our democracy ought to possess. And further, to arrange

the material so that the boy or girl who is forced to leave the high school at the end of one or two years may nevertheless get a better understanding and appreciation of some of the finer things of life so that he may enjoy himself as he goes along. For the student that remains, it is believed a general course will be saving of time of at least one year and that his ultimate conception of the entire field will be more thorough and fundamental. (Reeve, 1920, p. 265)

Our methods of teaching mathematics also need to be improved. The knowledge we now possess of individual differences in ability should make the study of mathematics a kind of laboratory course, in which more effective work can be done because the material can be better fitted to the individual pupil's needs. Such an arrangment of material decreases the need for so many reviews because each subject is kept in more or less constant use. As a result there is a gain in mathematical power and less need for an unreasonable amount of home study. In the first year it is even possible to get along with little or no home work. (Reeve, 1920, p. 266)

Although the change may represent more a subtle shift in emphasis than a blatant reversal of position, the social efficiency (and the developmentalist) rhetoric that Reeve had used to advocate his general mathematics program actually came--for whatever reason--to be reflected in the program itself. On the surface, this may seem perfectly reasonable; perhaps it was not an unintended consequence at all. Reeve did respond to his audience, and it may be that he made a reasoned and conscious compromise in order to implement some form of his program. However, if one

compares the rigorous-mathematics roots of the general-mathematics program with its ultimate "remedial" connotation (recognizing that it may not have had this connotation for all people who used the term), it seems quite reasonable to suggest that not all consequences were originally intended. Furthermore, one must consider the extent to which such a compromise had long-term consequences reflected in the "crisis" discussions of the 1930s.

Clearly, all that was occurring was not the simple effect of an easily determined cause; there was a complex interaction of people, ideas, and events. In the case of general mathematics, it is not simply a story of one mathematics educator's ideas about general mathematics undergoing a change. Many educators--influenced differently by such developed and developing traditions as Eliot's idea of individual differences in interest, the mental measurement educators' focus on individual differences in ability, the attacks on the usefulness of the study of mathematics, and the turn-of-the-century discussions about correlated (or unified) mathematics--had different and conflicting things to say about general mathematics. The crucial point for this paper is that the acceptance and use of a particular language of reform carries with it multiple consequences, some of which may be problematic. The fact is that, given its previous position of stature, the place of mathematics in the school curriculum was not as strong as it had once been. Furthermore, the developmentalists and, particularly, the social efficiency educators played a major role in calling the place of mathematics in the school curriculum into question. Clearly, Reeve and other mathematics educators who attempted to use the rhetoric of the critics to their advantage cared a great

deal about mathematics education, generally, and about unified mathematics, specifically. Given their era and the fact that unified mathematics, while prominent in discussions, did not seem to be prominent in classrooms, their response was not unreasonable. However, as one looks at what a number of mathematics educators were beginning to perceive as a growing "crisis" in mathematics education, one must at least bring up the possiblity that the acceptance of the rhetoric of the "outside" critics without fully considering the long-term consequences of such an action might have played a role in exacerbating the "crisis" that mathematics educators were beginning to perceive. It may be that, in any dispute, the language used to describe a situation provides a particular kind of lens for people to look through in attempting to make decisions about what should be done. Reeve was not unique in his use of the language of the developmentalists and social efficiency educators in his discussions; he was, however, somewhat unique in the extent to which he accepted criticisms and attempted to use them to further his own ideas. One could not claim that this attitude "caused" a crisis in mathematics education. At the very least, however, it did not hurt the cause of the developmentalists and social efficiency educators.

Ironically for people like Reeve who attempted to "use" David Snedden's criticisms of mathematics education to support changes, Snedden scorned most attempts at compromise. Snedden was not just a critic of "traditional" education; he was also critical of attempts at education for social efficiency which did not go far enough in achieving the social efficiency ideal (as his criticism

of the Cardinal Principles report mentioned earlier shows). About one of the "attempts to preserve old prescriptions in a new guise"--"composite mathematics"--he said:

> It is, for example, now proposed by some that a year's composite course in mathematics should be prescribed in the 9th grade in order to insure each high school entrant opportunity to "find himself" mathematically--and this after the pupil is expected to have given eight years already in part to the study of arithmetic! But is not this too much time to require for the purposes of guidance? In fact, more fundamentally, is any school subject, as now taught, at all well adpated sic to the ends of guidance? Must not the approaches eventually be on a wholly new basis? "Cut and try," "the expensive school of experience," "trial and error," "blind empiricism"--these are all heritages of prescientific stages in social evolution. When the military service can put into practice somewhat reasonably economical and effective "selective service," educators will surely not be found lagging. (Snedden, 1919b, p. 136)

IV

Because all historical examples are, to a certain extent, unique and because history does not directly tell us what to do in the present, one must be especially careful in attempting to use "the lessons of history." To further complicate matters in terms of this discussion, the crisis that existed in mathematics education before World War II was different in a very important way from our present "crisis" in education. In the years leading

up to the Second World War, mathematics educators began to sense a crisis; and, in the developmentalist/social efficiency coalition that had formed, they had clear and real opponents. Today, mathematics educators, "general" educators, and the general public seem to perceive a crisis in all of education; most people seem to be rallying around the idea of higher standards and increased requirements. The interesting--and perhaps problematic--coalition that is forming now between humanists and social efficiency educators is much like the coalition that formed in the late 1950s.

Recognizing these problems--that history cannot dictate present actions and that today's crisis is at a different "stage of development" than the crisis that existed before World War II--I believe that the historical example of using external reform rhetoric to advocate internal reform does offer some valuable perspective for us.

A more recent example of using external reform rhetoric in this way can be found in An Agenda for Action: Recommendations for School Mathematics of the 1980s (NCTM, 1980), the important and highly publicized statement presented by the National Council of Teachers of Mathematics.

The NCTM's first and foremost recommendation is that "problem solving be the focus of school mathematics in the 1980s" (p.1). In arguing for this recommendation, the authors of the Agenda attempt to characterize problem solving as a vital "basic skill." While clearly expressing some criticism of the "back-to-basics movement" (p.6), the authors also express sympathy for the idea of "teaching basic skills with greater effectiveness" (p.5) and

attempt to use the rhetoric of back-to-basics reformers to advocate a focus on problem solving.

Although the back-to-basics movement reflects a certain amount of diversity of ideas, many back-to-basics reformers would appear to be present-day manifestations of the social efficiency educators. Given the fact that the social efficiency educators' directly functional rationale for teaching mathematics has a long and controversial history, accepting the rhetoric of the back-to-basics reformers has potentially problematic consequences for mathematics educators.

Generally, when people talk about basic skills in mathematics, they are referring to computational skills with whole numbers, fractions, and decimals. Evidence for this is the fact that most of the minimum competency testing programs around the country have operationally defined basic skills in mathematics in this way. (As of 1980, at least thirty-eight states had mandated minimum competency testing programs. See Pipho, 1980.) Although the back-to-basics movement draws much of its strength from the general public, there are related events occurring within the professional education community. Much teacher effectiveness research is based on basic skills performance of students as reflected in standardized achievement tests (see, e.g., Good & Grouws, 1977, 1979). The influential publication of the National Institute of Education, Time to Learn (Denhan & Lieberman, 1980), also focuses on student acquisition of basic skills. And Gage (1977), in The Scientific Basis of the Art of Teaching, forms his conclusions by synthesizing the results of many studies with the same limited achievement focus.

The authors of the Agenda challenge the common meaning of "basic skills" and accept the view expressed by the National Council of Supervisors of Mathematics (1977). Ten basic skill areas are listed in the Agenda, including problem solving; applying mathematics in everyday situations; alertness to the reasonableness of results; estimation and approximation; appropriate computational skills; geometry; measurement; reading, interpreting, and constructing tables, charts, and graphs; using mathematics to predict; and computer literacy (NCTM, 1980, pp. 6-7). Essentially, the NCTM is saying that it, too, supports the emphasis on basic skills, as long as everything it wants in a mathematics program is called a "basic skill."

Placing the problem-solving recommendation within the context of back-to-basics rhetoric (i.e., classifying problem solving as simply one of ten basic skill areas) makes it difficult to respond to someone like Thomas Good (1979), who, using a more limited definition of basic skills (one that does not include the NCTM's view of problem solving), states: "The first step may be to improve basic skills. There is no reason to be excessively defensive about increasing students' performance in national achievement tests" (p.61). If problem solving is only one of ten basic skills to be learned, there is nothing wrong with wanting students to focus first on developing skill in the fundamental operations with whole numbers, fractions, and decimals. According to this view, the skill of problem solving can be taught after the fundamental operations are mastered. This clearly violates the spirit of the NCTM's problem-solving recommendation, yet the

acceptance of the language of "basic skills" limits the NCTM's response to this view.

Dewey (1910, 1933) and Hart (1978) point out the negative consequences of an exclusive and limited focus on skills. Clearly, and perhaps particularly in mathematics, not just what skills should be learned but the way such skills should be taught is crucial; that is, there is a danger that students will conceive of skills as ends in themselves, rather than as means to achieve other ends. As Dewey (1910) said, there is a "danger in those studies where the main emphasis is upon acquisition of skill":

> The tendency is to take the shortest cuts possible to gain the required end. This makes the subjects mechanical, and thus restrictive of intellectual power. In the mastery of reading, writing, drawing, laboratory technique, etc., the need of economy of time and material, of neatness and accuracy, of promptness and uniformity, is so great that these things tend to become ends in themselves, irrespective of their influence upon general mental attitude. Sheer imitation, dictation of steps to be taken, mechanical drill, may give results most quickly and yet strengthen traits likely to be fatal to reflective power. The pupil is enjoined to do this and that specified thing, with no knowledge of any reason except that by doing so he gets his result most speedily; his mistakes are pointed out and corrected for him; he is kept at pure repetition of certain acts till they become automatic. Later, teachers wonder why the pupil reads with so little expression, and figures with so little intelligent consideration of the terms of his

problem. . . . This method reduces the "training" of human beings to the level of animal. Practical skill, modes of effective technique, can be intelligently, nonmechanically used only when intelligence has played a part in their acquisition. (pp. 51-52)

It is clear that the authors of the Agenda would agree with Dewey; in fact, their recommendation that problem solving be the focus of school mathematics indicates this agreement that skills must be acquired with intelligence in order to be used with intelligence. However, the NCTM's argument is contradictory; or, at the very least, the NCTM diminishes its argument for permeating the mathematics curriculum with problem solving by characterizing what appears to be a general approach to the teaching and learning of mathematics as simply one of ten technical skills to be learned.

Acceptance of the language of "basic skills" has the potential, then, of distorting the problem-solving focus intended by the NCTM. The claim here is not that the NCTM would want its argument characterized in this way; the claim is that its use of back-to-basics rhetoric creates some problematic consequences that must be dealt with.

V

Whether or not we are actually in the midst of an educational crisis, the authors of recent reports--the most highly publicized of the reports being A Nation at Risk: The Imperative for Educational Reform--would have us believe that we are. What on the surface would seem to be a strong and consistent call for higher standards in education (for students and teachers) is, in

my opinion, a mixture of competing--and sometimes contradictory--claims. Just one example is the interesting phenomenon of many advocates of minimum competency testing, who are viewed by some as part of the cause of the crisis, seeing themselves as part of its resolution--though they speak now of establishing "standards" rather than "minimum competencies."

Despite the competing and contradictory claims that will frustrate us as we attempt to deal with this felt crisis, we have no reasonable choice but to deal with it. Because of those competing and contradictory claims, we need to be careful about the reform rhetoric we use.

References

Austin, C. M. (1921). The National Council of Teachers of Mathematics. Mathematics Teacher, 14, 1-4.

Bell, E. T. (1935). Review of The poetry of mathematics and other essays. American Mathematical Monthly, 42, 558-562.

Betz, W. (1908). The teaching of geometry in its relation to the present educational trend. School Science and Mathematics, 8, 625-633.

Betz, W. (1913). What mathematical subjects should be introduced in the curriculum of the secondary school? Mathematics Teacher, 5, 218-233.

Betz, W. (1930). Whither algebra?--A challenge and a plea. Mathematics Teacher, 23, 104-125.

Betz, W. (1932). A message from the new president of the National Council. Mathematics Teacher, 25, 300-302.

Breslich, E. R. (1916). Forward movements in secondary mathematics. School Review, 24, 283-297.

Breslich, E. R. (1920). Correlation of mathematical subjects. School Science and Mathematics, 20, 125-134.

Breslich, E. R. (1933). Secondary school mathematics and the changing curriculum. Mathematics Teacher, 26, 327-349.

Breslich, E. R. (1935). Mathematics for grades seven, eight, and nine. School Science and Mathematics, 35, 526-536.

Breslich, E. R., Schorling, R., Wright, H. C., & Irwin, H. N. (1916). Course of study in secondary mathematics in the University High School, the University of Chicago. School Review, 24, 648-674.

Denham, C., & Lieberman, A. (Eds.). (1980). Time to learn. Washington, DC: National Institute of Education.

Dewey, J. (1910). How we think. Boston: D. C. Heath & Co.

Dewey, J. (1933). How we thingk: A restatement of the relation of reflective thinking to the educative process. Boston: D. C. Heath & Co.

Douglass, H. R. (1937). Let's face the facts. Mathematics Teacher, 30, 56-62.

Gage, N. L. (1977). The scientific basis of the art of teaching New York: Teachers College Press.

Glover, I. R. (1934). Mathematics: A forgotten subject in present-day schools. Mathematics Teacher, 27, 199-204.

Good, T. L. (1979). Teacher effectiveness in the elementary school. Journal of Teacher Education, 30, 52-64.

Good, T., & Grouws, D. (1977). Teaching effects: Process-product study in fourth-grade mathematics classrooms. Journal of Teacher Education, 28, 49-54.

Good, T., & Grouws, D. (1979). The Missouri mathematics effectiveness project: An experimental study of fourth-grade classrooms. Journal of Educational Psychology, 71, 355-362.

Greenwood, J. M. (1894). Conference report on mathematics. Education, 15, 65-74.

Hart, W A. (1978). Against skills. Oxford Review of Education, 4, 205-216.

Hart, W. W. (1935). The need for a reorientation of secondary mathematics from the college viewpoint. Mathematics Teacher, 28, 69-79.

Hartung, M. L. (1935). The need for a reorientation of secondary school mathematics from the high school view point. Mathematics Teacher, 28, 80-90.

Hawkes, H. E. (1911). Educational values in mathematical teaching Mathematics Teacher, 4, 75-76, 85-89.

Hedrick, E. R. (1936). Crises in economics, education, and mathematics. Mathematics Teacher, 29, 109-114.

Jones, P. S. (1970). Epilogue: Summary and forecast. In The thirty-second yearbook of the National Council of Teachers of Mathematics. A history of mathematics education in the United States and Canada (pp. 451-465). Washington, DC: NCTM.

Kempner, A. (1938). On the need for cooperation between high school and college teachers of mathematics. Mathematics Teacher, 31, 117-123.

Kliebard, H. M. (1981, April). Education at the turn of the century: A crucible for curriculum change. Division B Invited Address, annual meeting of the American Educational Research Association, Los Angeles.

Lennes, N. J. (1909). Modern tendencies in the teaching of algebra. Mathematics Teacher, 1, 94-104.

Moore, E. H. (1926). On the foundations of mathematics. In The first yearbook of the National Council of Teachers of Mathematics. A general survey of progress in the last twenty-five years. No place of publication listed: NCTM. (Original work published 1902)

Morrison, H. C. (1914). Reconstructed mathematics in the high school: The adaptation of instruction to the needs, interests, and capacities of students. In The thirteenth yearbook of the National Society for the Study of Education: Part I. Bloomington, Illinois: Public School Publishing Company.

Myers, G. W. (1907). A class of content-problems for high school algebra. School Science and Mathematics, 7, 19-33.

Myers, G. W. (1918). Progressive teaching of mathematics--What is it? School Science and Mathematics, 18, 387-396.

Myers, G. W. (1921). Outstanding pedagogical principles now functioning in high-school mathematics. Mathematics Teacher, 14, 57-63.

Myers, G. W. (1922). Reaction vs. radicalism in the teaching of mathematics. Mathematics Teacher, 15, 137-146.

National Commission on Excellence in Education. (1983). A nation at risk: The imperative for educational reform. Washington, DC: U.S. Government Printing Office.

National Council of Supervisors of Mathematics. (1977). Position paper on basic mathematical skills. Arithmetic Teacher, 25, 19-22.

National Council of Teachers of Mathematics. (1931). The sixth yearbook of the National Council of Teachers of Mathematics. Mathematics in modern life. New York: Bureau of Publications, Teachers College, Columbia University.

National Council of Teachers of Mathematics. (1936). The eleventh yearbook of the National Council of Teachers of Mathematics. Mathematics in modern education. New York: Bureau of Publications, Teachers College, Columbia University.

National Council of Teachers of Mathematics. (1940). The fifteenth yearbook of the National Council of Teachers of Mathematics. The place of mathematics in secondary education. New York: Bureau of Publications, Teachers College, Columbia University.

National Council of Teachers of Mathematics. (1980). An agenda for action: Recommendations for school mathematics of the 1980s. Reston, VA: Author.

National Educational Association. (1893). Report of the Committee on Secondary School Studies. Washington, DC: U.S. Government Printing Office.

National Educational Association. (1899). Report of the Committee on College Entrance Requirements. Chicago: Author.

National Education Association. (1918). Cardinal principles of secondary education. Washington, DC: U.S. Government Printing Office.

Perry, J. (Ed.). (1902). Discussion on the teaching of mathematics. New York: The Macmillan Company.

Pipho, C. (1980). Analysis of state minimum competency testing programs. Denver, CO: Education Commission of the States.

Reeve, W. D. (1916). Unification of mathematics in the high school. School and Society, 4, 203-212.

Reeve, W. D. (1920). General mathematics for the high school: Its purpose and content. Educational Administration and Supervision, 6, 258-273.

Reeve, W. D. (1922). The case for general mathematics. Mathematics Teacher, 15, 381-391.

Reeve, W. D. (1924). The general trend of mathematics education in secondary schools. Mathematics Teacher, 17, 449-458.

Schorling, R. (1928). Suggestions for the solution of an important problem that has arisen in the last quarter of a century. In The first yearbook of the National Council of Teachers of Mathematics. A general survey of progress in the last twenty-five years. No place of publication listed: NCTM.

Seerly, H. H. (1894). The report of the Committee of Ten: The mathematical conference. Education, 15, 239-241.

Sigurdson, S. E. (1962). The development of the idea of unified mathematics in the secondary school curriculum 1890-1930 (Doctoral dissertation, University of Wisconsin-Madison, 1962). Dissertation Abstracts International, 23 (16), 1997. (University Microfilms Order No. 63-00616)

Smith, D. E. (1905). Movements in mathematical teaching. School Science and Mathematics, 5, 134-139.

Smith, D. E. (1909). The teaching of arithmetic. New York: Teachers College, Columbia University.

Smith, D. E., & McMurry, F. M. (1903). Mathematics in the elementary school. Teachers College Record, 4, 91-160.

Smith, E. R. (1927). The mathematics curriculum in the senior high school. In The second yearbook of the National Council of Teachers of Mathematics. Curriculum problems in teaching Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1927.

Smith, E. R. (1929). Mathematics in the school of the future. Teachers College Record, 30, 795-807.

Snedden, D. (1919a). Cardinal principles of secondary education. School and Society, 9, 517-527.

Snedden, D. (1919b). Mathematics in the education of youth from 12 to 18 years of age--Some proposals for further study. Educational Administration and Supervision, 5, 125-144.

Young, J. W. A. (1903). What is the laboratory method? Mathematical Supplement of School Science, 1, 50-56.

Young, J. W. A. (1906). The teaching of mathematics in the elementary and the secondary school. New York: Longmans, Green, and Company.

Young, J. W. A. (1907). My struggle with the Italian language and the morals I drew from it for the teaching of mathematics. School Review, 15, 225-260.

Young, J. W. A. (1912). Concerning systematic exposition of mathematics and the fusion of its various branches in secondary instruction. School Science and Mathematics, 12, 447-456.